

Exercise 43

Find the critical numbers of the function.

$$f(x) = x^2 e^{-3x}$$

Solution

A critical number is a value of x for which the derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^2 e^{-3x}) \\ &= \left[\frac{d}{dx}(x^2) \right] e^{-3x} + x^2 \left[\frac{d}{dx}(e^{-3x}) \right] \\ &= (2x)e^{-3x} + x^2 \left[(e^{-3x}) \cdot \frac{d}{dx}(-3x) \right] \\ &= 2xe^{-3x} + x^2[(e^{-3x}) \cdot (-3)] \\ &= 2xe^{-3x} - 3x^2 e^{-3x} \\ &= xe^{-3x}(2 - 3x) \end{aligned}$$

Set $f'(x) = 0$ and solve for x .

$$\begin{aligned} xe^{-3x}(2 - 3x) &= 0 \\ x = 0 \quad \text{or} \quad 2 - 3x &= 0 \\ x = 0 \quad \text{or} \quad x &= \frac{2}{3} \end{aligned}$$